THE INFLUENCE OF POWER LEVEL ON THE CHARACTERISTICS OF ACOUSTIC ATTENUATION IN $BI_{12}GEO_{20}$ CRYSTALS

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ABSTRACT

$Bi_{12}GeO_{20}$ and $Bi_{12}SiO_{20}$ crystals are very similar in their properties, in particular, have a very low attenuation of longitudinal and shear acoustic waves, which is untypical for heavy metal oxides. A unique feature of these crystals is abnormal temperature dependencies of attenuation. But data on acoustic attenuation and its frequency dependence in these crystals are different in different sources. Ill-defined power levels of acoustic wave used for the measurement may be a root of this difference.

Due to elastic nonlinearity inherent in real media, part of acoustic power generates high-order harmonics, which is equivalent to increase in acoustic attenuation for the fundamental mode. Elastic nonlinearity manifests itself to different degrees, depending on crystal properties, frequency band and, for most part, on power level entered into the medium. Usually, spatial power distributions for the fundamental and higher acoustic modes are very complex, and they are difficult to approximate.

The optical method for visualizing the structure of acoustic wave, called the "Schlieren image method", was used for the measurements.

Spatial distributions of acoustic waves (images) for different power levels and frequencies are presented. A very close to linear dependence of acoustic attenuation for longitudinal modes on input power level is found for $Bi_{12}GeO_{20}$ crystals.

KEYWORDS: Elastic nonlinearity, acoustic attenuation.

1. THEORETICAL DISCUSSIONS

In reality, the assumption and conclusions of the linear theory of elasticity should take into account nonlinear components in the equation of state of an elastic medium for high-frequency elastic waves with finite amplitudes.

The medium state with elastic non-linearity and dynamic strain is described by the generalized Hook's law.

$$T_i = c_{ij}(s_j)\cdot s_j, \qquad i, j = 1,2,3\,, \tag{1}$$

Herein, T_i, s_j are the components of elastic stress and strain, respectively, $c_{ij}(s)$ are the components of medium's elastic constants, which are assumed to be constant under the linear assumption, but in reality they depend on the elastic strain, if medium's non-linear properties are taken into account.

Using the coupled mode method [Ref. 1,2], it is possible to compose a system of differential equations describing the features of acoustic modes traveling in an elastic non-linear medium. The non-linear dependence results in the fact that an initially harmonic wave, while traveling in the medium, changes its shape, approaching a saw-tooth shape. As such deformed wave shape equals to a sum of harmonics, depletion of fundamental harmonics power occurs during a non-linear process that generates high-order harmonics. The generation of such high-order acoustic harmonics results in increased attenuation of the process, as the attenuation coefficient increases proportionally to the square of frequency. Thus, elastic non-linearity in a medium damps acoustic waves, even if there are no other sources of losses.

The amplitude increment dS_1 of the fundamental acoustic wave along the path dx may be written as

$$dS_1 = -(\alpha_1 + \beta_1 \cdot S_1)\cdot S_1 \cdot dx\,. \tag{2}$$

It is assumed that the negative increment is proportional to the amplitude value in an arbitrary point $S_1(x)$ with the coefficient consisting of two components: α_1 and $\beta_1 \cdot S_1$. The first coefficient α_1 characterizes natural attenuation in the medium (equivalent to the friction or viscosity coefficient), and the second coefficient $\beta_1 \cdot S_1$ determines the process of energy swap to higher harmonics due to the non-linear interaction.

In this differential equation, the physically proved assumption that the component determining the power swap is proportional to the amplitude of elastic wave s_1. Equation 2 may be solved for any degree of approximation for this component, the solution being $\beta_1 S_1^n$.

The coefficient β_1 is an empiric constant, which depends on elastic and non-linear properties of the medium (its second and third-order elastic constants), and on process rate (frequency of acoustic wave), too.

Under the initial condition $S_1(0) = S_0$, Equation 2 may be solved in the following form:

$$S_1(x) = S_0 \frac{\alpha_1}{(\alpha_1 + \beta_1 \cdot S_0)\cdot \exp(\alpha_1 \cdot x) - \beta_1 \cdot S_0}\,, \tag{3}$$

It is significant that, if the dissipative losses are excluded $(\alpha_1 = 0)$, the depletion of fundamental acoustic mode due to non-linear effects is determined by a hyperbolic dependence:

$$S_1(x) = S_0 \frac{1}{1 + \beta_1 S_0 x}\,.$$

2. EXPERIMENTAL MEASUREMENTS

Fig. 1 shows dependencies of acoustic spatial distributions calculated with depletion and attenuation under different initial conditions. Points in this figure show experimental data for the longitudinal acoustic mode in the $NaBi(MoO_4)_2$ crystal under several acoustic power levels [Ref. 2]. For this case, $\alpha_1 = 0{,}037$ 1/cm (0,32 dB/cm), $\beta_1 \approx 0{,}11 \cdot 10^5$ 1/cm $(\beta_1 S_0) \cong 0{,}2$ for P_a=6 W/cm^2 at f_1=270 MHz.

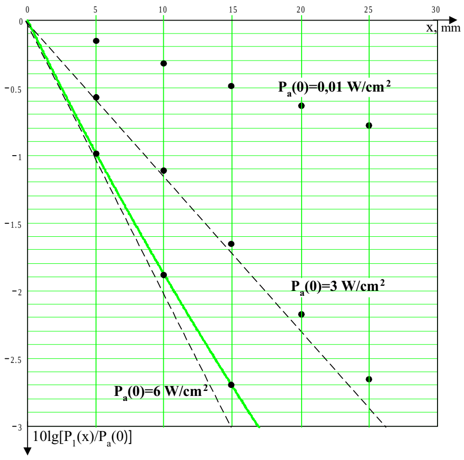


Fig. 1
Spatial distributions of fundamental acoustic longitudinal wave in the [001] $NaBi(MoO_4)_2$ crystal under several initial power levels

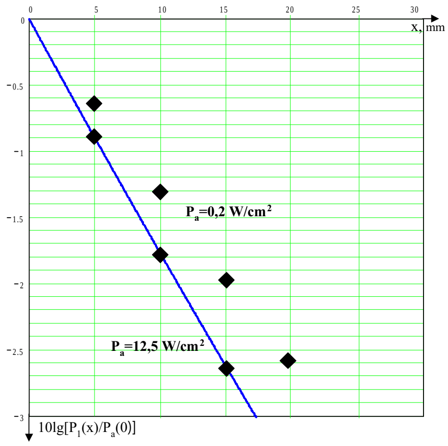


Fig. 2
Spatial distributions of fundamental acoustic longitudinal wave in the [111] $Bi_{12}GeO_{20}$ crystal under several initial power levels

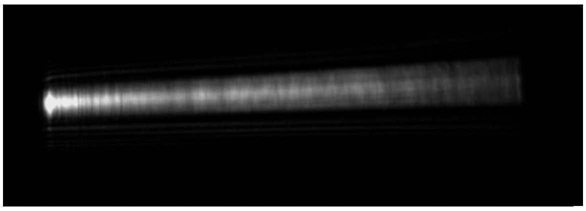
The $Bi_{12}GeO_{20}$ and $Bi_{12}SiO_{20}$ crystals belong to the cubic class [Ref. 3]. They have a number of unique acoustic, piezoelectric, and optical properties. An abnormal thermal dependence of acoustic attenuation is known for these crystals. Also are typical an unusual combination of slow acoustic velocities and relatively low level of attenuation, strong piezoelectric effect and possibility for non-resonant acoustic wave generation, high optical quality and photorefractive properties. Nevertheless, the attention characteristics for these crystals published in the literature are fragmentary and contradicting.

Fig. 2 shows the spatial distributions for the fundamental harmonics power at the frequency of f=233 MHz in the [111] $Bi_{12}GeO_{20}$ crystal under several initial power levels.

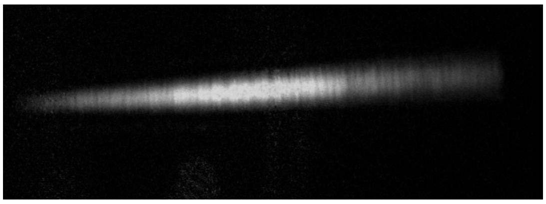
Experimental data show that α_1=0,15 1/cm, β_1=0,02*10^5 1/cm under P_a=12,5 W/cm^2, S_0=3,0*10^{-5} (elastic strain at the generation point),

Fig. 3 shows the Schlieren image illustrating the longitudinal acoustic wave traveling in the [111] direction in the $Bi_{12}GeO_{20}$ crystal at the frequency f=370 MHz, the piezoelectric transducer height being $h \approx 0.5$ mm.

Fig. 3 also shows the Schlieren image for the second diffraction order visualizing the power distribution for the second harmonics generated due to the non-linear interaction, the initial power of the fundamental harmonics being 0.25 w.



Fundamental mode



Second harmonics

Fig. 3
Schlieren images of acoustic harmonics in the $Bi_{12}GeO_{20}$ crystal (L-mode, f=370 MHz)

The parameters of the second acoustic harmonics generated in the non-linear medium may be found by solving the differential equation composed using the coupled mode method:

$$\frac{dS_2}{dx} = -\alpha_2 \cdot S_2 + \beta_2 \cdot S_1^2, \qquad (4)$$

In this expression, two counteractive processes determine the increment of the second mode dS_2: the process of acoustic attenuation $(-\alpha_2 \cdot S_2)$ and generation $(\beta_2 \cdot S_1^2)$ due to the quadratic non-linear dependence in the medium's equation of state.

If the functional dependence $S_1(x)$ is already found, this equation may be solved numerically.

If the attenuation of the fundamental mode is low, a simple approximation $S_1(x) \approx S_0 \cdot \exp[-(\alpha_1 + \beta_1 \cdot S_0)x]$ may be used, and thus a closed-form solution for the amplitude distribution of the second harmonics $S_2(x)$ may be found:

$$S_2(x) = \frac{\beta_2 \cdot S_0^2}{\alpha_2 - 2(\alpha_1 + \beta_1 \cdot S_0)} \cdot \left[\exp(-2(\alpha_1 + \beta_1 \cdot S_0)x) - \exp(-\alpha_2 x)\right], \qquad (5)$$

The coefficients β_1 and β_2 characterize the effects of the same non-linear process but in different acoustic modes, but the relation between these coefficients is quite complex. Therefore, the constants of elastic nonlinearity can be experimentally found while acoustic attenuation is being measured under different acoustic power levels.

Fig. 4 shows the spatial representation of the amplitude for the second harmonics traveling along the [111] direction (v = 3,1*10^5 cm/sec). Frequency dependence of acoustic attenuation is assumed to be $\alpha(n\omega) = n^{2,22} \cdot \alpha(\omega)$

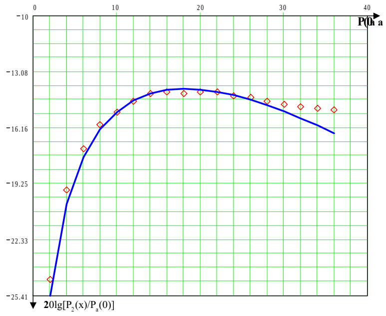


Fig. 4
Spatial distributions of the second acoustic mode along the [111] direction in the $Bi_{12}GeO_{20}$ crystal (f=230 MHz)

The material constant of non-linearity β does not depend on the measurement conditions and is characterized by the elastic constants of the medium.

$$|\beta| \cong \frac{2}{\pi}\beta_2\lambda_a,$$

$$\beta = -\left(3 + \frac{c_{ef}^{(3)}}{c_{ef}^{(2)}}\right)$$

Here λ_a is the wavelength of the fundamental acoustic harmonics, $c_{ef}^{(2)}, c_{ef}^{(2)}$ are the effective elastic constants of the second and third order, respectively, for the selected crystal direction.

The parameters of the $Bi_{12}GeO_{20}$ crystal are presented in the Table. For comparison, it also shows the parameters of $NaBi(MoO_4)_2$ crystal, which non-linear parameters are very strong.

CONCLUSIONS

$Bi_{12}GeO_{20}$ crystals have rather mild elastic non-linearity. But for high-frequency acousto-optic and acoustoelectronic applications, this non-linearity may affect the performance of such devices. Especially important are cross-product components, which level restricts system dynamic range.

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Table

Parameters of the $Bi_{12}GeO_{20}$ and $NaBi(MoO_4)_2$ crystals

Media	**Acoustic mode and direction**	**$V*10^{-5}$, cm/c**	**f_1, MHz**	**α, dB/cm**	**$(\alpha_1+\beta_1 S_0)$, dB/cm**
$Bi_{12}GeO_{20}$	L [111]	3,1	232	1,38	1,52
$NaBi(MoO_4)_2$	L [001]	4,0	275	0,32	1,90
Media	**P_a, W/cm^2**	**S_0*10^5**	**β_1, 1/cm**	**β_2, 1/cm**	**$\vert\beta\vert$**
$Bi_{12}GeO_{20}$	12,5	3	$0{,}2*10^4$	10^4	8,6
$NaBi(MoO_4)_2$	6	1,8	$1{,}1*10^4$	$6{,}1*10^4$	50